

## Linear Sum

### *Deduction of general formula*

The sum of all integers in the interval [a,b], where {a,b} are integers

$$\begin{aligned}
 \sum_{i=a}^b i &= \sum_{i=b}^a i = \frac{1}{2} \left( \left( \sum_{i=a}^b i \right) + \left( \sum_{i=b}^a i \right) \right) = \frac{1}{2} \left( \left( \sum_{i=0}^{b-a} i + a \right) + \left( \sum_{i=0}^{a-b} i + b \right) \right) = \\
 &= \frac{1}{2} \left( \left( \sum_{i=0}^{b-a} i + a \right) + \left( \sum_{i=0}^{b-a} -i + b \right) \right) = \frac{1}{2} \sum_{i=0}^{b-a} ((i+a) + (-i+b)) = \\
 &= \frac{1}{2} \sum_{i=0}^{b-a} (a+b) = \frac{(|b-a|+1)(a+b)}{2}
 \end{aligned}$$

$$\sum_{i=a}^b i = \frac{(|b-a|+1)(a+b)}{2}$$